# Filters

* In this part we will discuss filters design and implementation

## * Ideal filter Response

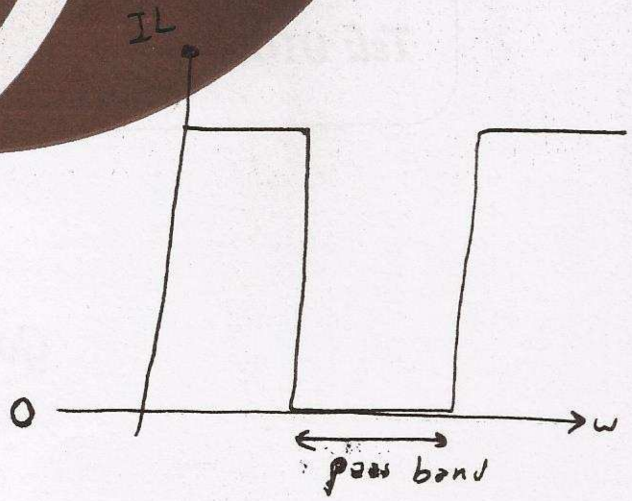
$P_{LR} \triangleq$ power loss ratio

$$= \frac{\text{Power available from source}}{\text{Power absorbed by the load}}$$

$$\boxed{P_{LR} = \frac{1}{1 - |\Gamma(\omega)|^2}}$$ حفظ

$IL \triangleq$ The insertion loss

$$\boxed{IL = 10 \log_{10} P_{LR}}$$




Ideal response

## * Filter Design techniques:

1. Periodic structures

2. Image Parameter Method

3. Insertion loss Method

## * Filter Implementation techniques:

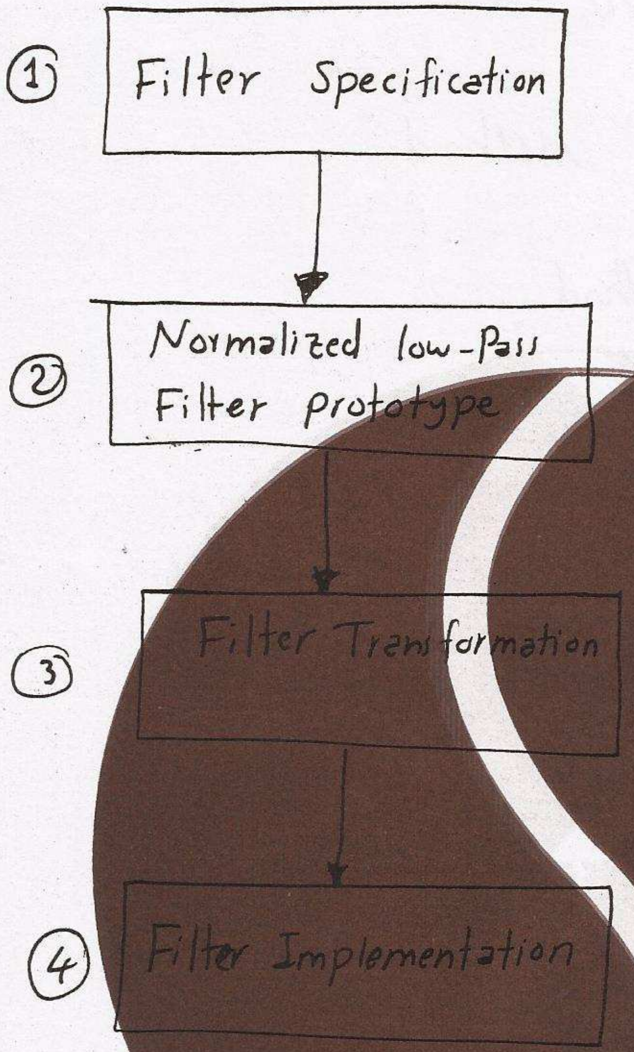
1. stubs with separating unit Elements

2. stepped impedance low-Pass Filter

3. Coupled line filters

4. Coupled Resonator Filters.

## * The flow chart of Realization Procedure :



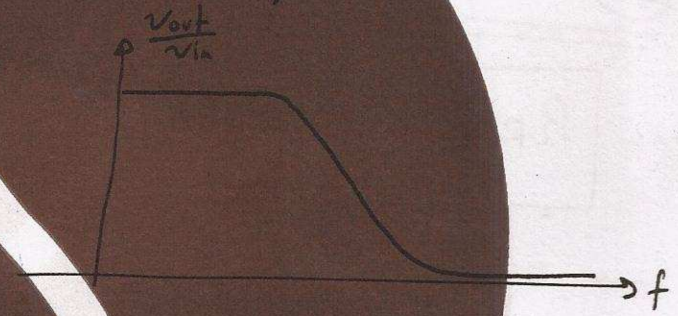
## I The Filter Specifications

$$\therefore \quad P_{LR} = \frac{1}{1 - |\Gamma(\omega)|^2} = f(\omega^2)$$

by equating the power loss ratio with different polynomial, we can obtain different filter response

### I Binomial "Butterworth" "Maximally flat"

$$\boxed{P_{LR} = 1 + K^2 \left(\frac{\omega}{\omega_c}\right)^{2N}}$$



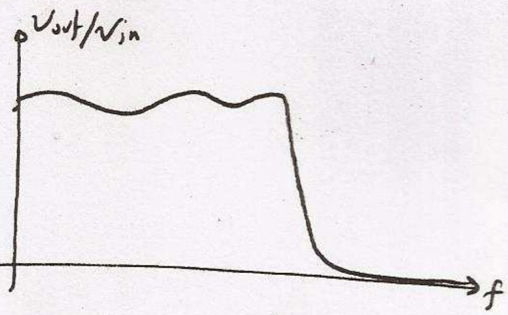

→ maximally flat PB

→ IL in SB "stop band" = 20 N dB/decade

→ slow transition

### II Chebychev type 1

$$\boxed{P_{LR} = 1 + K^2 \, T_N^2\left(\frac{\omega}{\omega_c}\right)}$$



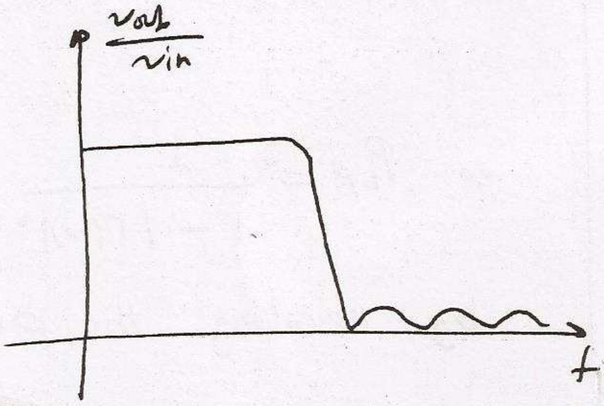

→ equal ripple in PB

→ IL in SB 20N dB/decade "highest IL in SB"

→ sharp transition

## (III) Inverse chebychev " chebychev type (2) "

$$\boxed{P_{LR} = 1 + \frac{K^2}{T_N^2\left(\frac{w_c}{w}\right)}}$$



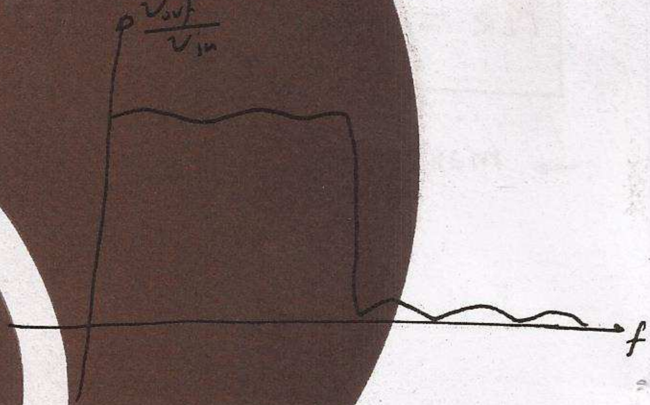

→ flat in PB

→ Equal ripple in SB

→ sharp Transition

## (IV) Elliptic

$$\boxed{P_{LR} = 1 + K^2 R_N^2\left(\frac{w}{w_c}\right)}$$

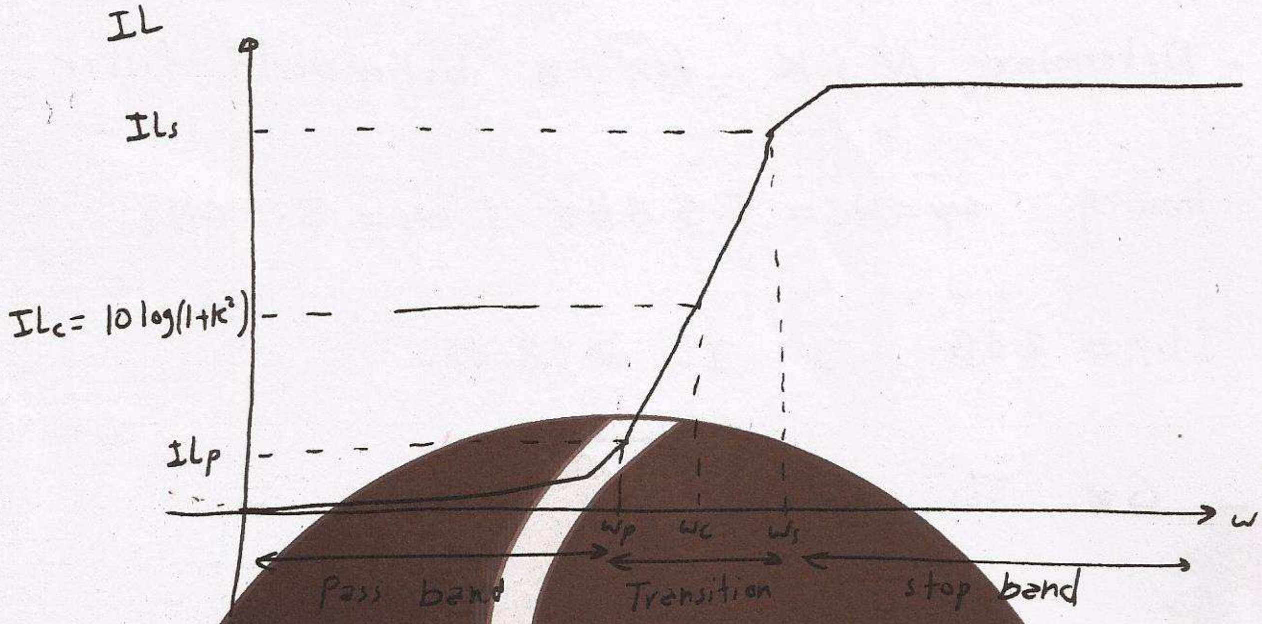



→ equal ripple in PB

→ equal ripple in SB

→ sharpest transition

## * low pass filter prototype response




Given ILp , ILs ⟶ we need to calculate

K , N [ N ≡ the filter order ]

There are two Methods to obtain N, K

[1] Analitically

[2] Graphically

## Example

- Determine $N$, $K$ for a butterworth filter having $w_p = w_c = 5.5\ GHz$, $w_s = 8.9\ GHz$

$IL_p = 2\ dB$, $IL_s \geq 18\ dB$

- Get $IL_s$ for your filter

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(I) We will use the analytical solution

$$\boxed{P_{LR} = 1 + K^2 \left(\frac{w}{w_c}\right)^{2N}} \longrightarrow \text{Binomial}$$

* $IL_p = 10 \log P_{LR}\big|_{w = w_p = w_c} = 2\ dB$

$$IL_p = 10 \log (1 + K^2) = 2\ dB$$

$$1 + K^2 = 10^{0.2}$$

$$\boxed{K = \sqrt{10^{0.2} - 1} = 0.764783}$$

* $IL_s = 10 \log P_{LR}\big|_{w = w_s} \geq 18\ dB$

$$10 \log \left[1 + K^2 \left(\frac{w_s}{w_c}\right)^{2N}\right] \geq 18 \text{ dB}$$

$$\left[1 + K^2 \left(\frac{w_s}{w_c}\right)^{2N}\right] \geq 10^{1.8}$$

$$K^2 \left(\frac{w_s}{w_c}\right)^{2N} \geq 10^{1.8} - 1$$

$$\left(\frac{w_s}{w_c}\right)^{2N} \geq \left[\frac{10^{1.8} - 1}{K^2}\right]$$

$$2N \ln\left(\frac{w_s}{w_c}\right) \geq \ln\left[\frac{10^{1.8} - 1}{K^2}\right]$$

$$N \geq \frac{\ln\left[(10^{1.8} - 1)/K^2\right]}{2 \ln(w_s/w_c)}$$

$$N \geq 4.84622$$

$$\boxed{N = 5} \leftarrow \text{next integer}$$

لاحظ اذا ذكر في السؤال $IL = 18$ dB

ده المقصود $IL \geq 18$ dB أن نحاول تحقيق القيمة

المطلوب أو أكثر منها

لأن الـ (filter order) يجب أن يكون integer

$$\therefore IL_s = 10 \log \left[ 1 + K^2 \left( \frac{\omega_1}{\omega_c} \right)^{2N} \right]$$

$$= 10 \log \left[ 1 + (0.764783)^2 * \left( \frac{8.9}{5.5} \right)^{10} \right]$$

$$\boxed{IL_s = 18.633399 \ dB}$$

∴ قمنا بتحقيق IL اكبر من المطلوب

## Example

- Determine $N, K$ for a butter worth filter having $\omega_c = 5.5$ GHz , $\omega_s = 8.9$ GHz

$IL_s = 18$ dB

لاحظ في الأسئلة السابقة تم إعطاء معلومتين $IL_p$ و $IL_s$

حتى نحصل على ال $K$ وال $N$

أما في هذا المثال فإنه لدينا معلومة واحدة فقط $IL_s = 18$ dB

في هذه الحالة نفترض دائماً أن هناك معلومة أخرى $\boxed{IL_c = 3 \text{ dB}}$

$$\Rightarrow \quad IL_c = 10 \log \left[ 1 + K^2 \left(\frac{\omega_c}{\omega_c}\right)^{2N} \right] = 3$$

$$= 10 \log \left[ 1 + K^2 \right] = 3$$

$$\Rightarrow \quad \boxed{K = 1}$$

$$IL_s = 10 \log \left[ 1 + \left(\frac{\omega_s}{\omega_c}\right)^{2N} \right] = 18$$

$$1 + \left(\frac{\omega_s}{\omega_c}\right)^{2N} = 10^{1.8}$$

$$2N \ln\left(\frac{\omega_s}{\omega_c}\right) = \ln\left[10^{1.8} - 1\right]$$

$$N = \frac{\ln[10^{1.8}-1]}{2\ln(ws/wc)} = 4.289$$

$$\boxed{N = 5}$$

$\boxed{\text{Analytical solution}}$ كل ما سبق كان ال

## (II) Graphical solution

- يتم استخدام مجموعة من ال curves لايجاد ال filter order (N)
- كل نوع filter له curves مختلفة

* Bionomial "Butterworth" → has its own curves
* Chebyshev type 1 → has its own curves

وهكذا

← لاحظ كل نوع filter يتم رسمه اكثر من مره حسب قيمة ال K

فمثلا ال curves في الصفحة التالي ← خاصة بحالة

① Butterworth ② $K = 1$ (3 dB at $w = wc$)

Low-Pass Filter Prototype

# Design Curves for Maximally Flat LPF

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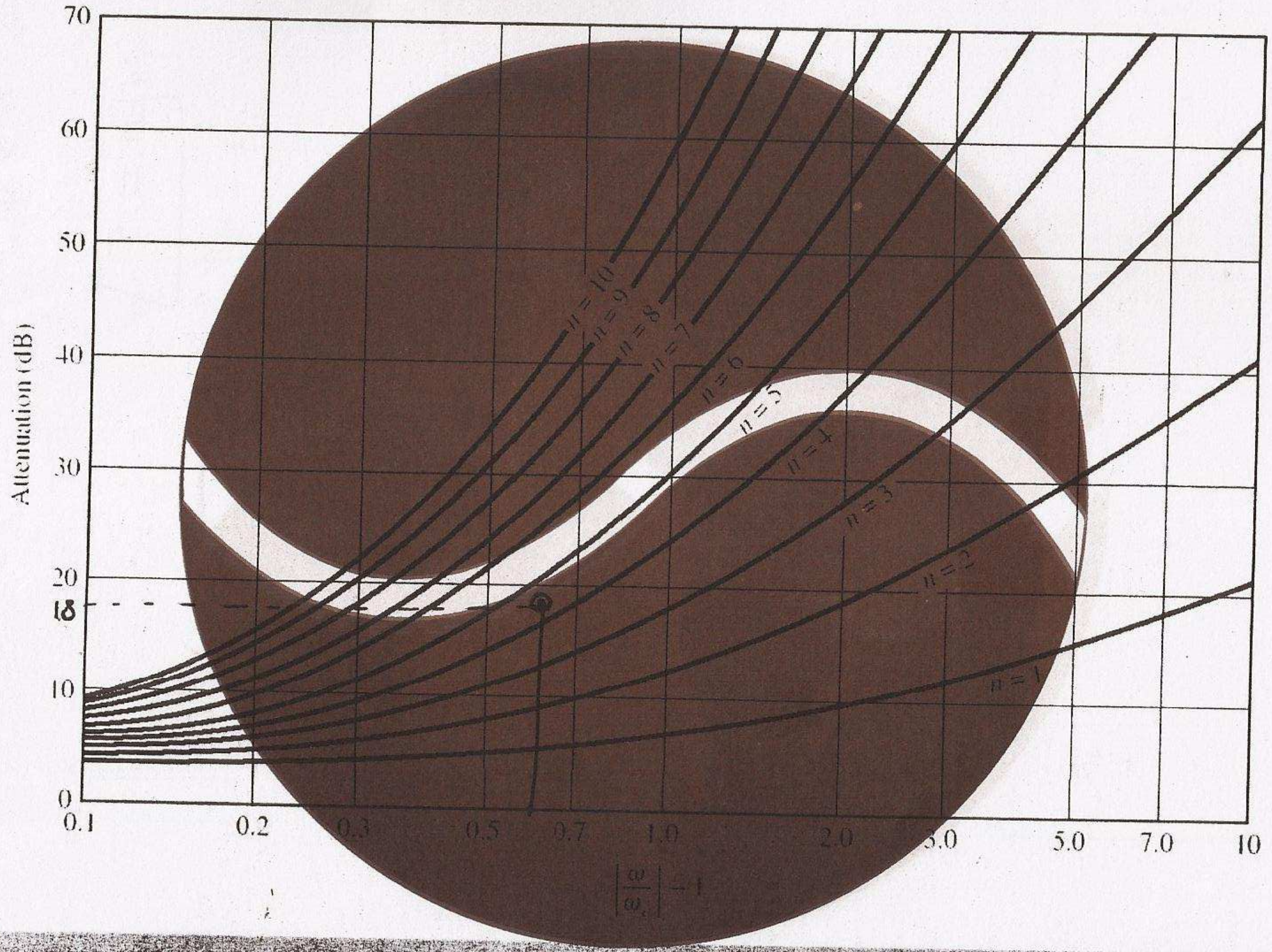


Given the attenuation (insertion loss) at a frequency in the stopband frequency, the order of the filter N can be determined.

As indicated in the previous graph

$$\therefore \left|\frac{\omega_s}{\omega_c}\right| - 1 = \frac{8.9}{5.5} - 1 = \underline{\underline{0.618}}$$

$$\Rightarrow n > 4$$

$$\therefore \boxed{n = 5}$$

## 2 Normalized low-Pass filter Prototype

* Given the filter order $N$

* and the filter type [ Butterworth , chebychev . . . ]

* the value of $K$

→ We need to get the value of the capacitors and inductors used in the prototype

* There are two types of solutions

1. Analytical

2. Using tables

Hint → to get the protype capacitors and inductors value we allways assume the following assumptions

1. $R_s = 1$  2. $w_c = 1$

## Example

Get the prototype capacitors and inductor values for a Butterworth filter of $2^{nd}$ order

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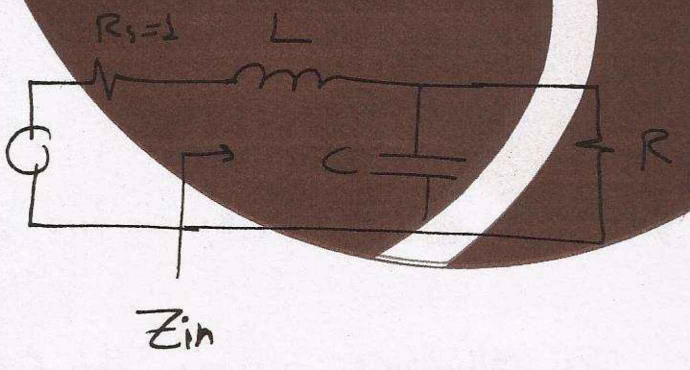
### (I) Analytical solution

$\boxed{K=1}$ ⟸ لاحظ يتم اعطاء قيمة K

→ for butterworth only

∴ $2^{nd}$ order filter ⟹ $N=2$

∴ The prototype can be plotted as follow




$$\boxed{Z_{in} = jwL + \left(\frac{1}{jwC} // R\right)}$$